PHYS 100C, Lecture 16

Tuesday, May 26, 2009
4:09 PM

2^{nd} Newton's Law:

$$F = \frac{\partial p}{\partial t} \quad \text{or}$$

using work: $W \equiv \int F \cdot dl = \int F \cdot u \cdot dt$

$$W = \int \frac{\partial p}{\partial t} \cdot u \cdot dt$$

$$\frac{\partial p}{\partial t} = \frac{\partial}{\partial t}\left(\frac{m\bar{u}}{(1-u^2/c^2)^{1/2}}\right) = \frac{m\frac{\partial \bar{u}}{\partial t}}{(1-u^2/c^2)^{1/2}} -$$

$$-\frac{m}{2}\cdot\frac{\left(-2u\cdot\frac{\partial u}{\partial t}\right)\cdot\bar{u}}{(1-u^2/c^2)^{3/2}} = \frac{m\left(\frac{\partial \bar{u}}{\partial t} - \cancel{\frac{u^2}{c^2}\cdot\frac{\partial \bar{u}}{\partial t}} + \cancel{\frac{u^2}{c^2}\cdot\frac{\partial \bar{u}}{\partial t}}\right)}{(1-u^2/c^2)^{3/2}} =$$

$$= \frac{m\,\partial u/\partial t}{(1-u^2/c^2)^{3/2}}$$

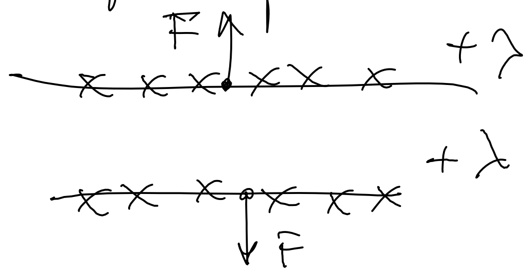
Note that when defined:

$$E = \frac{mc^2}{(1-u^2/c^2)^{1/2}} \Rightarrow \frac{\partial E}{\partial t} = \frac{m u\cdot\frac{\partial u}{\partial t}}{(1-u^2/c^2)^{3/2}} =$$

$$= \frac{\partial p}{\partial t}\cdot u$$

Big revelation:
Magnetism is relativistic form of electricity.

Thought experiment: two lines of charge will repell each other. No B-field.



Now get into a frame moving with velocity v.
Now charge density is

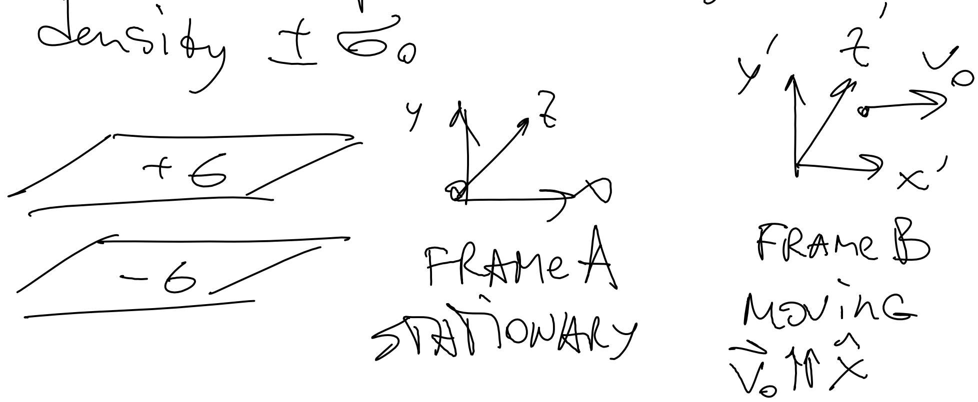
$$\lambda^* = \gamma \cdot \lambda > \lambda$$

increase due to Lorentz contraction.

Coulomb repulsion is stronger as we go faster!

To account for changes, we have to include ~~MAGN.~~ field (wasn't there in stationary frame of reference).

Consider capacitor charge density $\pm \sigma_0$



In A: $E_y^A = \frac{\sigma_0}{\epsilon_0}$ $E_x = E_z = 0$

$V_0 \parallel x$

In A: $E_y^A = \frac{\sigma_0}{\varepsilon_0}$ $\quad E_x = E_z = 0$

In B: $E_y^B = \frac{\sigma}{\varepsilon_0}$ $\quad$ where $\sigma = \gamma \cdot \sigma_0$

since charge density increases due to Lorentz contraction.

$E_x^B = E_z^B = 0$, still.

In B we have also MAGN. field due to currents:

$$\vec{K}_\pm = \pm \sigma V_0 \cdot \hat{x} = \pm \gamma_0 \sigma_0 V_0 \cdot \hat{x}$$

$$B_z^B = -\mu_0 \gamma_0 \sigma V_0$$

$(B^A = 0)$ $\qquad \gamma_0 = \frac{1}{(1 - V_0^2/c^2)^{1/2}}$

How do arbitrary fields $\vec{E}$, $\vec{B}$ transform from one FRAME to another?

In FRAME B we have $\vec{E}, \vec{B}$.

Introduce frame C, moving with v w.r. to B and $u = \frac{v + V_0}{1 + \frac{v V_0}{c^2}}$ w.r. to A

Consider frame C relative to A, fields transform:

$$E_y^C = \frac{\sigma^C}{\varepsilon_0} = \frac{\gamma\sigma_0}{\varepsilon_0}$$

where $\gamma = \dfrac{1}{(1-u^2/c^2)^{1/2}}$

Also

$$B_z^C = -\mu_0\gamma\sigma_0 \cdot u$$

and: $\dfrac{E_y^C}{E_y^B} = \dfrac{\gamma}{\gamma_0}$

$$\frac{\gamma}{\gamma_0} = \left(\frac{1-v_0^2/c^2}{1-u^2/c^2}\right)^{1/2} = \left[\frac{c^2-v_0^2}{c^2-\left(\frac{v_0+v}{1+\frac{v_0 v}{c^2}}\right)^2}\right]^{1/2} =$$

{a few steps skipped}

$$= \frac{1+vv_0/c^2}{\sqrt{1-v^2/c^2}} = \gamma\left(1+\frac{vv_0}{c^2}\right)$$

Note that $v_0 = -\dfrac{B_z^B}{\gamma\mu_0\sigma_0}$

$$E_y^C = \gamma\left(1-\frac{B_z^B \cdot v}{\gamma\mu_0\sigma_0}\cdot\frac{1}{c^2}\right)\cdot\underbrace{\frac{\gamma\sigma_0}{\varepsilon_0}}_{E_y^B} =$$

$$= \gamma\left(E_y^B - \frac{v B_z^B}{\underbrace{\mu_0 \varepsilon_0 c^2}_{=1}}\right) = \gamma\left(E_y^B - v B_z^B\right)$$

Similarly:

$$B_z^C = -\frac{\gamma}{\gamma_0} \cdot \mu_0 \gamma_0 u = \gamma\left(1 + \frac{v v_0}{c^2}\right) \mu_0 \gamma \sigma_0 u =$$

$$= \gamma B_z^B - \gamma \underbrace{\mu_0 \varepsilon_0}_{1/c^2} v E_y^B = \gamma\left(B_z^B - \frac{v}{c^2} E_y^B\right)$$

Similarly for E_z / B_y
(rotate xy into xz capacitor):

$$E_z^C = \gamma\left(E_z^B + v B_y^B\right)$$

$$B_y^C = \gamma\left(B_z^B + \frac{v}{c^2} E_z^B\right)$$

Parallel to motion (x-axis):

$$E_x^B = E_x^C = E_x^A$$

(capacitor planes in yz produces the same $\sigma^?$ and has no change of E_x).

Magnetic field stays the same too: $B_x^B = B_x^C = B_x$

* Bottom line message:
E, B fields $\perp$ to motion

get transformed into each other,
but not $E, B \parallel$ to motion.
pure E in one F.O.R. can become
E and B in another F.O.R. etc.